Universität Rostock

Traditio et Innovatio

LeMoS

Lecture 7:

Introduction into Finite Volume Method I

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Transformation of the Navier-Stokes equations in the Finite Volume Method

$$\frac{\partial V_i}{\partial t} + \frac{\partial (V_i V_J)}{\partial x_j} = F_i - \frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial}{\partial x_j}\left(\frac{\partial}{\partial x_j} V_i\right)$$

$$\int_U \left[\frac{\partial V_i}{\partial t} + \frac{\partial (V_i V_J)}{\partial x_j}\right] dU = \int_U \left[F_i - \frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial}{\partial x_j}\left(\frac{\partial}{\partial x_j} V_i\right)\right] dU$$

$$\frac{\partial}{\partial t}\int_U V_i dU + \int_S V_i \vec{V}\vec{n} dS = \int_U F_i dU - \frac{1}{\rho}\int_S p\vec{e}_i \vec{n} dS + \nu \int_S grad V_i \vec{n} dS$$

$$\int_S \vec{V}\vec{n} dS = 0$$

Simple case (without viscous term)

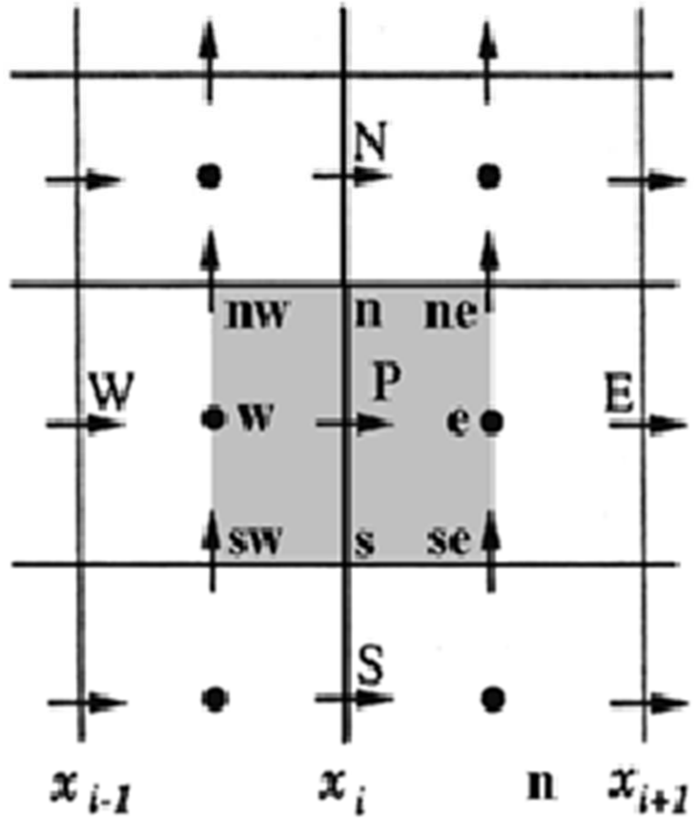
$$\begin{cases} \dfrac{\partial u_i}{\partial t} + \dfrac{\partial u_i u_j}{\partial x_j} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x_i} \\ \dfrac{\partial u_j}{\partial x_j} = 0 \end{cases}$$

$$\frac{\partial}{\partial t}\int_U u_i dU + \int_S u_i \vec{u}\vec{n} dS = -\frac{1}{\rho}\int_S p\vec{e}_i \vec{n} dS$$

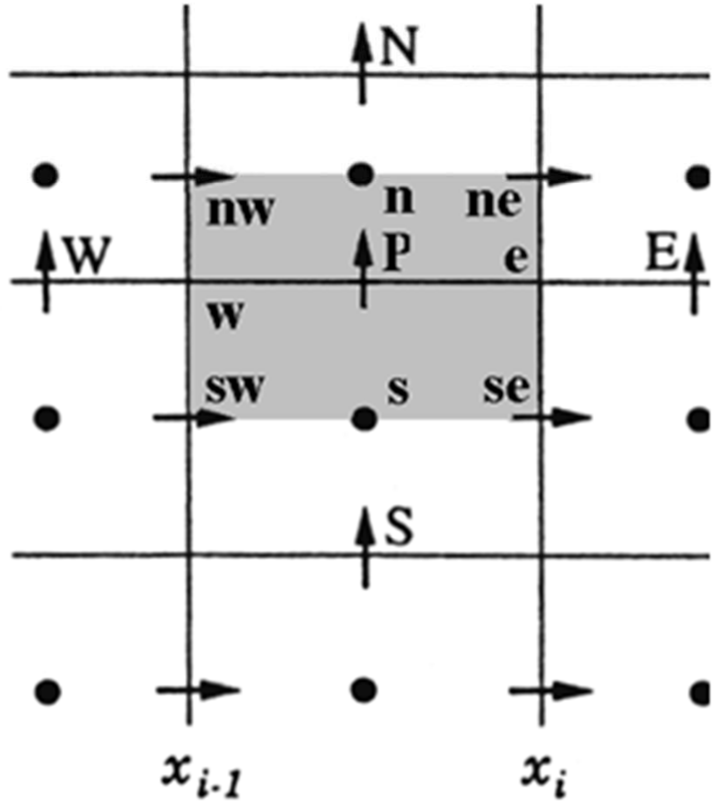
$\rho=1$

Simple case: staggered grid

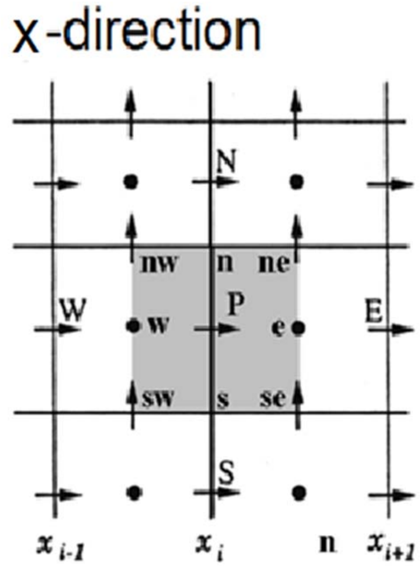
x-direction



y-direction



Pressure and unsteady terms



Unsteady term:

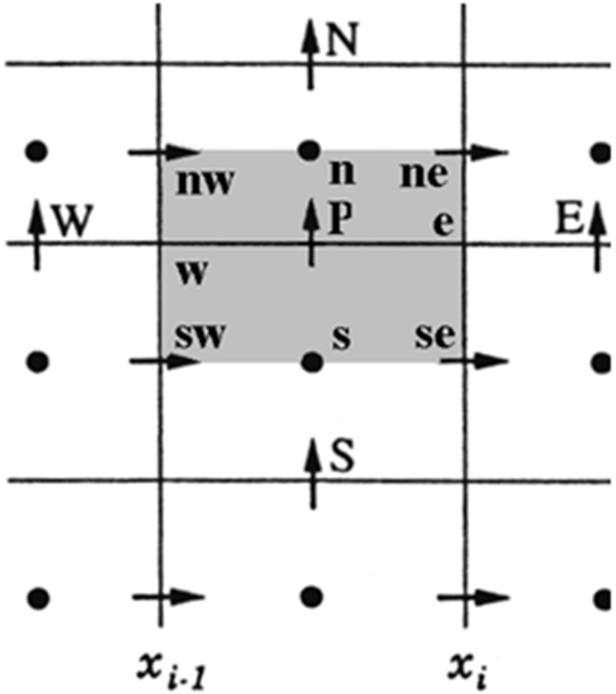
$$\frac{\partial}{\partial t}\int_U u_x dU = \Delta^2 \frac{(u^{n+1}_{xi+1j} + u^{n+1}_{xij}) - (u^{n}_{xi+1j} + u^{n}_{xij})}{2\Delta t}$$

Source (pressure) term:

$$Q_1^p = -\int_S p\vec{e}_1\vec{n}dS \approx -(p_e S_e - p_w S_w) \;\; = -(p_{i+1j} - p_{ij})\Delta$$

Pressure and unsteady terms

y-direction



Unsteady term:

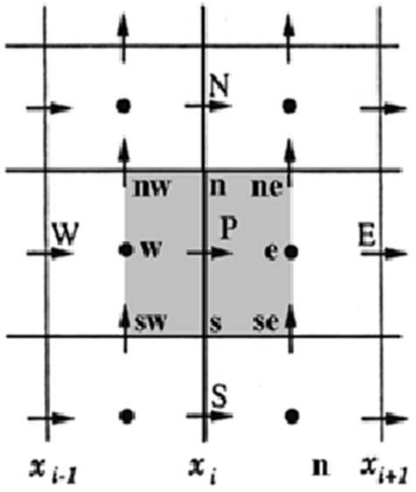
$$\frac{\partial}{\partial t}\int_U u_y dU = \Delta^2 \frac{(u_{yij+1}^{n+1} + u_{yij}^{n+1}) - (u_{yij+1}^{n} + u_{yij}^{n})}{2\Delta t}$$

Pressure term:

$$Q_2^p = -\int_S p\vec{e}_2\vec{n}dS \approx -(p_n S_n - p_s S_s) \;\; = -(p_{ij+1} - p_{ij})\Delta$$

Convection term: x component

x-direction



$$\int_S u_x \vec{u}\vec{n} dS =$$

$$(u_{xe}u_{xe} - u_{xw}u_{xw} + u_{xn}u_{yn} - u_{xs}u_{ys})\Delta$$

$$u_{xe} = u_{xi+1j}, u_{xw} = u_{xij}, u_{xn} = \frac{1}{4}(u_{xij+1} + u_{xij} + u_{xi+1j} + u_{xi+1j+1}),$$

$$u_{xs} = \frac{1}{4}(u_{xij-1} + u_{xij} + u_{xi+1j} + u_{xi+1j-1}).$$

$$= \Delta[u_{xi+1j}u_{xi+1j} - u_{xij}u_{xij} +$$

$$\frac{1}{4}(u_{xij+1} + u_{xij} + u_{xi+1j} + u_{xi+1j+1})\frac{1}{4}(u_{yij+1} + u_{yij} + u_{yi+1j} + u_{yi+1j+1}) -$$

$$\frac{1}{4}(u_{xij-1} + u_{xij} + u_{xi+1j} + u_{xi+1j-1})\frac{1}{4}(u_{yij-1} + u_{yij} + u_{yi+1j} + u_{yi+1j-1})] =$$

Convection term: x component

$$a^c_{xi+1\,j}u_{xi+1\,j} + a^c_{xij}u_{xij} + a^c_{xij+1}u_{xij+1} + a^c_{xi+1\,j+1}u_{xi+1\,j+1} + a^c_{xij-1}u_{xij-1} + a^c_{xi+1\,j-1}u_{xi+1\,j-1}.$$

$$a^c_{xi+1\,j} = \Delta(u_{xi+1\,j} + \frac{1}{16}(u_{yij+1} + u_{yi+1\,j+1} - u_{yij-1} - u_{yi+1\,j-1}))$$

$$a^c_{xij} = \Delta(-u_{xij} + \frac{1}{16}(u_{yij+1} + u_{yi+1\,j+1} - u_{yij-1} - u_{yi+1\,j-1}))$$

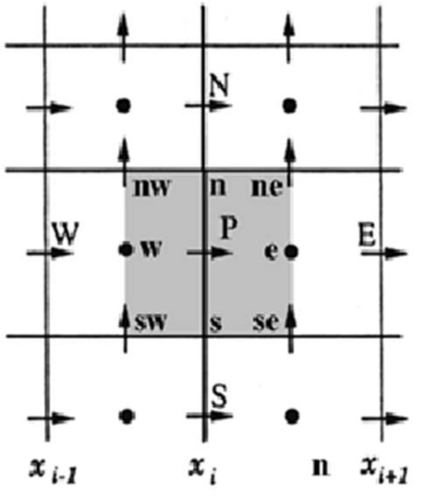
$$a^c_{xij+1} = \frac{\Delta}{16}(u_{yij+1} + u_{yij} + u_{yi+1\,j} + u_{yi+1\,j+1})$$

$$a^c_{xi+1\,j+1} = \frac{\Delta}{16}(u_{yij+1} + u_{yij} + u_{yi+1\,j} + u_{yi+1\,j+1})$$

$$a^c_{xij-1} = -\frac{\Delta}{16}(u_{yij-1} + u_{yij} + u_{yi+1\,j} + u_{yi+1\,j-1})$$

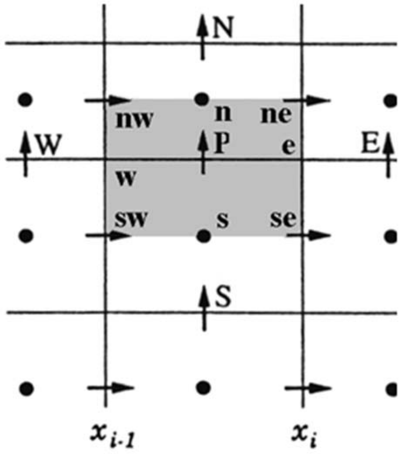
$$a^c_{xi+1\,j-1} = -\frac{\Delta}{16}(u_{yij-1} + u_{yij} + u_{yi+1\,j} + u_{yi+1\,j-1})$$

x-direction



Convection term: y component

y-direction



$$\int_S u_y \vec{u}\vec{n} dS = (u_{ye}u_{xe} - u_{yw}u_{xw} + u_{yn}u_{yn} - u_{ys}u_{ys})\Delta$$

$$u_{yn} = u_{yij+1}, u_{ys} = u_{yij}, u_{ye} = \frac{1}{4}(u_{yij} + u_{yi+1j} + u_{yij+1} + u_{yi+1j+1}),$$

$$u_{yw} = \frac{1}{4}(u_{yij} + u_{yi-1j} + u_{yij+1} + u_{yi-1j+1}).$$

$$= \Delta[\frac{1}{4}(u_{yij} + u_{yi+1j} + u_{yij+1} + u_{yi+1j+1})\frac{1}{4}(u_{xij} + u_{xi+1j} + u_{xij+1} + u_{xi+1j+1}) -$$

$$\frac{1}{4}(u_{yij} + u_{yi-1j} + u_{yij+1} + u_{yi-1j+1})\frac{1}{4}(u_{xij} + u_{xi-1j} + u_{xij+1} + u_{xi-1j+1}) +$$

$$u_{yij+1}u_{yij+1} - u_{yij}u_{yij}] =$$

Convection term: y component

$$a^c_{yi+1j}u_{yi+1j} + a^c_{yij}u_{yij} + a^c_{yij+1}u_{yij+1} + a^c_{yi+1j+1}u_{yi+1j+1} + a^c_{yi-1j}u_{yi-1j} + a^c_{yi-1j+1}u_{yi-1j+1}.$$

$$a^c_{yi+1j} = \frac{\Delta}{16}(u_{xij} + u_{xi+1j} + u_{xij+1} + u_{xi+1j+1})$$

$$a^c_{yij} = \Delta(-u_{yij} + \frac{1}{16}(u_{xi+1j} + u_{xi+1j+1} - u_{xi-1j} - u_{xi-1j+1}))$$

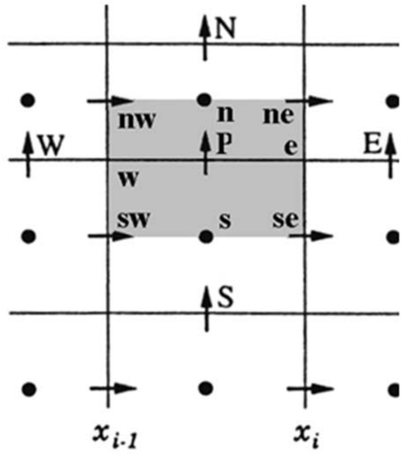
$$a^c_{yij+1} = \Delta(\frac{1}{16}(u_{xi+1j} + u_{xi+1j+1} - u_{xi-1j} - u_{xi-1j+1}) + u_{yij+1})$$

$$a^c_{yi+1j+1} = \frac{\Delta}{16}(u_{xij} + u_{xi+1j} + u_{xij+1} + u_{xi+1j+1})$$

$$a^c_{xi-1j} = -\frac{\Delta}{16}(u_{xij} + u_{xi-1j} + u_{yij+1} + u_{yi-1j+1})$$

$$a^c_{xi-1j+1} = -\frac{\Delta}{16}(u_{xij} + u_{xi-1j} + u_{yij+1} + u_{yi-1j+1})$$

y-direction



Discretization of equation

All terms together

$$\begin{cases} \Delta^2 \dfrac{(u^{n+1}_{xi+1j} + u^{n+1}_{xij}) - (u^{n}_{xi+1j} + u^{n}_{xij})}{2\Delta t} + \\ +a^c_{xi+1j}u_{xi+1j} + a^c_{xij}u_{xij} + a^c_{xij+1}u_{xij+1} + a^c_{xi+1j+1}u_{xi+1j+1} + a^c_{xij-1}u_{xij-1} + a^c_{xi+1j-1}u_{xi+1j-1} = -(p_{i+1j} - p_{ij})\Delta \\ \Delta^2 \dfrac{(u^{n+1}_{yij+1} + u^{n+1}_{yij}) - (u^{n}_{yij+1} + u^{n}_{yij})}{2\Delta t} + \\ +a^c_{yi+1j}u_{yi+1j} + a^c_{yij}u_{yij} + a^c_{yij+1}u_{yij+1} + a^c_{yi+1j+1}u_{yi+1j+1} + a^c_{yi-1j}u_{yi-1j} + a^c_{yi-1j+1}u_{yi-1j+1} = -(p_{ij+1} - p_{ij})\Delta \end{cases}$$

X-component

Y-component

Problem: which time slice to take on the r.h.s. of last equations?

First idea: n → fully explicit scheme. Disadvantage→ unstable

Second idea: n+1 → fully implicit. Disadvantage → nonlinear Equations, since „a“ coefficients depend on u.

Way out: fully implicit with simple iterations

Literature:

1. Kornev, N. & Cherunova I. (2014). ***Lectures on computational fluid dynamics and heat Transfer with applications to human thermodynamics.*** Bookboon Publisher.
2. Ferziger J. and Peric M. (2002) *Computational Methods for Fluid Dynamics*. Springer.

Thank you very much for your attention!